Show that the set of Turing Machines that accept only a finite number of strings is not Recursively Enumerable.

Proof. We reduce the complement of the halting language to this. Suppose we have an (M, w) pair. Build M' so that (a) M' accepts all strings of length 2 or less, and (b) if |x| > 2 M'(x) simulates M on w for |x| steps. If M halts on w within |x| steps M' accepts x; otherwise M' rejects x. If M does halt within n steps M' accepts all x with |x| >= n, which is an infinite set. If M does not halt on w M' accepts only the strings of length 2 or less, which is a finite set. If we could recognize if M' accepts only a finite set of string we could also recognize if M does not halt on w. But we can't.