Show that the set of Turing Machines that accept only a finite number of strings is not Recursively Enumerable.

Proof. We reduce the complement of the halting language to this. Suppose we have an ( $M, w$ ) pair. Build $\mathrm{M}^{\prime}$ so that (a) $\mathrm{M}^{\prime}$ accepts all strings of length 2 or less, and (b) if $|x|>2 \mathrm{M}^{\prime}(\mathrm{x})$ simulates M on w for $|x|$ steps. If $M$ halts on $w$ within $|x|$ steps $M^{\prime}$ accepts $x$; otherwise $M^{\prime}$ rejects $x$. If $M$ does halt within $n$ steps $M^{\prime}$ accepts all $x$ with $|x|>=n$, which is an infinite set. If $M$ does not halt on $w M^{\prime}$ accepts only the strings of length 2 or less, which is a finite set. If we could recognize if $\mathrm{M}^{\prime}$ accepts only a finite set of string we could also recognize if $M$ does not halt on $w$. But we can't.

